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Effective Theory of the Triton

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Abstract

We apply the effective field theory approach to the three-nucleon system. In particular, we consider $S = 1/2$ neutron-deuteron scattering and the triton. We show that in this channel a unique nonperturbative renormalization takes place which requires the introduction of a single three-body force at leading order. With one fitted parameter we find a good description of low-energy data. Invariance under the renormalization group explains some universal features of the three-nucleon system —such as the Thomas and Efimov effects and the Phillips line— and the origin of $SU(4)$ symmetry in nuclei.

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1 Introduction

Effective field theories (EFT's) are a powerful concept designed to explore a separation of scales in physical systems [1]. For example if the momenta k of two particles are much smaller than the inverse range of their interaction $1/R$, observables can be expanded in powers of kR . Following the early work of Weinberg and others [2], EFT's have become quite popular in nuclear physics [3, 4]. Their application, however, is complicated by the presence of shallow (quasi) bound states, which create a large scattering length $a \gg R$. In this fine-tuned case, a simple perturbative expansion in kR already breaks down at rather small momenta $k \sim 1/a$. In order to describe bound states with typical momenta $k \sim 1/a$, the range of the EFT has to be extended. A certain class of diagrams has to be resummed which generates a new expansion in kR where powers of ka are kept to all orders. For the two-nucleon system a power counting that incorporates this resummation has been found recently [5, 6, 7]. A number of two-nucleon observables have been studied in the low-energy theory where pions have been integrated out [8]. Furthermore, pions can be included explicitly and treated perturbatively in this scheme [6]. This approach has successfully been applied to NN scattering and deuteron physics [9].

The three-nucleon system is a natural test-ground for the understanding of nuclear forces that has been reached in the two-nucleon system. Furthermore, it shows some remarkable universal features. In the context of potential models, it has been found that different models of the two-nucleon interaction that are fitted to the same low-energy two-nucleon (NN) data predict different but correlated values of the triton binding energy B_3 and the S -wave nucleon-deuteron (Nd) scattering length $a_3^{(1/2)}$ in the spin $S = 1/2$ channel; all models fall on a line in the $B_3 \times a_3^{(1/2)}$ plane, the Phillips line [10]. Other universal features of three-body systems are the existence of a logarithmic spectrum of bound states that accumulates at zero energy as the two-particle scattering length a_2 increases (the Efimov effect [11]), and the collapse of the deepest bound state when the range of the two-body interaction r_2 goes to zero (the Thomas effect [12]). These universal features have no obvious explanation in the framework of conventional potential models. (Alternative explanations of these features that are likely equivalent to ours can be found in Ref. [13].)

The critical issue in the extension of any theory from the two- to the three-body sector is the relative size of three-body forces. A naive dimensional analysis argument that does not incorporate the fine tuning in the two-body sector suggests that three-body forces appear only in higher orders in the kR expansion. We have shown that this is supported by Nd data in the $S = 3/2$ channel [14]. However, for a system of three bosons a non-trivial renormalization enhances the size of three-body forces [15]. (For a different approach, see Ref. [16].) Here we will show that the renormalization of the three-nucleon system in the $S = 1/2$ channel is nonperturbative and requires a single three-body force at leading order. With such a force, the EFT allows to understand the above mentioned universal features in a unified way. Being $SU(4)$ symmetric, this force also supports the view of spin-flavor as an approximate symmetry of nuclei [17]. Moreover, if its single parameter is fitted to $a_3^{(1/2)}$, we find a good description of the energy dependence of scattering and

of triton binding.

2 Preliminaries

In order to avoid the difficulties due to the long-range Coulomb force, we concentrate here on the neutron-deuteron (nd) system. For simplicity, we also restrict ourselves to scattering below the deuteron breakup threshold where S -waves are dominant. There is only one low-energy scale, $\sqrt{MB_2} \approx 40$ MeV, where B_2 is the binding energy of the deuteron and M the nucleon mass. Since $\sqrt{MB_2}$ is small compared to the pion mass, the pions can be integrated out, only nucleons remaining in the EFT as explicit degrees of freedom. The leading order of this pionless theory [5, 6, 7] is equivalent to the leading order in the KSW counting scheme [6] where pions are included perturbatively. As a consequence, the extension of our results to higher energies is well defined.

There are two S -wave channels for neutron-deuteron scattering, corresponding to total spin $S = 3/2$ and $S = 1/2$. For scattering in the $S = 3/2$ channel all spins are aligned and the two-nucleon interactions are only in the 3S_1 partial wave. The two-body interaction is attractive but the Pauli principle forbids the three nucleons to be at the same point in space. As a consequence, this channel is insensitive to short-distance physics and precise predictions are obtained in a straightforward way [14]. There is also no three-body bound state in this channel. The $S = 1/2$ channel is more complicated. The two-nucleon interaction can take place either in the 3S_1 or in the 1S_0 partial waves. This leads to an attractive interaction which sustains a three-body bound state, the triton. The $S = 1/2$ channel also shows a strong sensitivity to short-distance physics as the Pauli principle does not apply. We will see that the generic features of this channel are thus very similar to the system of three spinless bosons. There is in the latter a strong cutoff dependence even though all Feynman diagrams are finite, and renormalization requires a leading-order three-body force counterterm [15].

Let us start from the assumption that the three-body force is of natural size. The lowest-order effective Lagrangian is then given by

$$\begin{aligned} \mathcal{L} = & N^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2M} \right) N - C_0^t \left(N^T \tau_2 \vec{\sigma} \sigma_2 N \right)^\dagger \cdot \left(N^T \tau_2 \vec{\sigma} \sigma_2 N \right) \\ & - C_0^s \left(N^T \sigma_2 \boldsymbol{\tau} \tau_2 N \right)^\dagger \cdot \left(N^T \sigma_2 \boldsymbol{\tau} \tau_2 N \right) + \dots, \end{aligned} \quad (1)$$

where the dots represent higher-order terms suppressed by derivatives and more nucleon fields. $\vec{\sigma}$ ($\boldsymbol{\tau}$) are Pauli matrices operating in spin (isospin) space, respectively. The contact terms proportional to C_0^t (C_0^s) correspond to two-nucleon interactions in the 3S_1 (1S_0) NN channels. Their renormalized values are related to the corresponding two-body scattering lengths a_2^t and a_2^s by $C_0^{s,t} = \pi a_2^{s,t}/2M$. Since no derivative interactions appear at this order, this Lagrangian generates only two-nucleon interactions of zero range. For practical purposes, it is convenient to rewrite this theory by introducing “dibaryon” fields with the quantum numbers of two nucleons [18]. In our case, we need two dibaryon

fields: (i) a field \vec{T} with spin (isospin) 1 (0) representing two nucleons interacting in the 3S_1 channel (the deuteron) and (ii) a field \mathbf{S} with spin (isospin) 0 (1) representing two nucleons interacting in the 1S_0 channel. Using a Gaussian path integration, it is straightforward to show that the Lagrangian (1) is equivalent to

$$\begin{aligned} \mathcal{L} = & N^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2M} \right) N + \Delta_T \vec{T}^\dagger \cdot \vec{T} + \Delta_S \mathbf{S}^\dagger \cdot \mathbf{S} \\ & - \frac{g_T}{2} \left(\vec{T}^\dagger \cdot N^T \tau_2 \vec{\sigma} \sigma_2 N + h.c. \right) - \frac{g_S}{2} \left(\mathbf{S}^\dagger \cdot N^T \sigma_2 \boldsymbol{\tau} \tau_2 N + h.c. \right) + \dots \end{aligned} \quad (2)$$

At first it may look like the Lagrangian (2) contains more parameters than the original one, Eq. (1). However, the scales Δ_T and Δ_S are arbitrary and included in Eq. (2) only to give the dibaryon fields the usual mass dimension of a heavy field. They can easily be removed by rescaling the dibaryon fields. All observables depend only on the ratios $g_{T,S}^2/\Delta_{T,S}$. Note that at this lowest order the EFT in the two-nucleon sector is formally equivalent to a certain limit of a separable potential, known as the Amado model [19]. We stress, however, that the introduction of “quasi-particle” fields \vec{T} and \mathbf{S} carries here *no* dynamical assumptions [5].

Since the theory is nonrelativistic, all particles propagate forward in time, the nucleon tadpoles vanish, and the propagator for the nucleon fields is

$$iS(p) = \frac{i}{p_0 - p^2/2M + i\epsilon}. \quad (3)$$

The dibaryon propagators are more complicated because of the coupling to two-nucleon states. The bare dibaryon propagator is simply a constant, $i/\Delta_{S,T}$, but the full propagator gets dressed by nucleon loops to all orders as illustrated in Fig. 1. The nucleon-loop

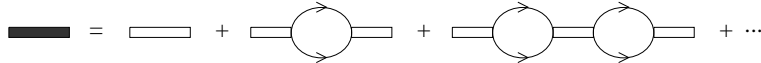


Figure 1: Dressing of the bare dibaryon propagator.

integral has a linear ultraviolet (UV) divergence which can be absorbed in $g_{T,S}^2/\Delta_{T,S}$, a finite piece determined by the unitarity cut, and subleading terms that have already been omitted in Eq. (2). Summing the resulting geometric series leads to

$$iD_{S,T}(p) = \frac{-i}{-\Delta_{S,T} + \frac{Mg_{S,T}^2}{2\pi} \sqrt{-Mp^0 + \vec{p}^2/4} - i\epsilon + i\epsilon}, \quad (4)$$

where $g_{T,S}$ and $\Delta_{T,S}$ now denote the renormalized parameters. The NN scattering amplitude in the respective channel is obtained by attaching external nucleon lines to the

dressed propagator. In the center-of-mass frame, the S -wave amplitude in the 1S_0 , 3S_1 channels for energy $E = k^2/M$ is

$$T^{s,t}(k) = \frac{4\pi}{M} \left(-\frac{2\pi\Delta_{S,T}}{Mg_{S,T}^2} - ik \right)^{-1}, \quad (5)$$

and the renormalized parameters $g_{T,S}$ and $\Delta_{T,S}$ can be determined from

$$a_2^{s,t} = \frac{Mg_{S,T}^2}{2\pi\Delta_{S,T}}. \quad (6)$$

Note that in this zero-range approximation,

$$\sqrt{MB_2} = 1/a_2^t, \quad (7)$$

which holds only approximately, the discrepancy coming from range corrections.

3 $S = 1/2$ nd Scattering and the Triton

We now use the Lagrangian, Eq. (2), to describe nd -scattering in the $S = 1/2$ channel and the triton. The two-nucleon interactions can take place both in the 3S_1 and 1S_0 partial waves. There are two coupled amplitudes, a and b , as the triton can be built by adding a neutron to a proton and a neutron which are in either of the two partial waves. The amplitude a which has both an incoming and outgoing dibaryon field \vec{T} gives the phase shifts for $S = 1/2$ nd scattering; a is coupled to the amplitude b which has an incoming dibaryon \vec{S} and an outgoing dibaryon \vec{T} . Although only a corresponds to elastic $S = 1/2$ nd scattering, both amplitudes have the quantum numbers of the triton. The diagrams for the two amplitudes are obtained from the Lagrangian (2). However, the leading piece of all diagrams for a (b) is of order Mg_T^2/Q^2 (Mg_Tg_S/Q^2), where Q stands for either the external momentum k or $1/a_2^{s,t}$. Therefore the diagrams have to be summed to all orders, which is conveniently done by solving the coupled integral equations shown in Fig. 2.

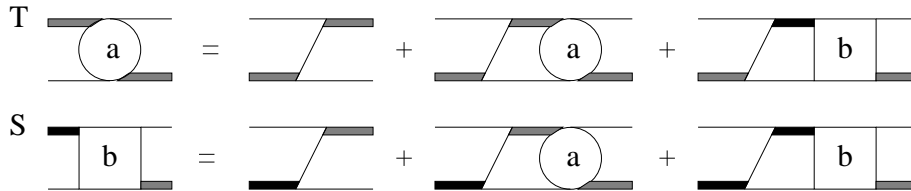


Figure 2: Coupled integral equations for $S = 1/2$ nd scattering. $\vec{T}(\vec{S})$ dibaryon is indicated by grey shaded (black) thick line.

After the integration over the time component of the loop-momentum and the projection onto the S -waves has been carried out, we have

$$\frac{3}{2} \left(1/a_2^t + \sqrt{3p^2/4 - ME} \right)^{-1} a(p, k) \quad (8)$$

$$= K(p, k) + \frac{2}{\pi} \int_0^\Lambda \frac{q^2 dq}{q^2 - k^2 - i\epsilon} K(p, q) [a(q, k) + 3b(q, k)]$$

$$2 \frac{\sqrt{3p^2/4 - ME} - 1/a_2^s}{p^2 - k^2} b(p, k) \quad (9)$$

$$= 3K(p, k) + \frac{2}{\pi} \int_0^\Lambda \frac{q^2 dq}{q^2 - k^2 - i\epsilon} K(p, q) [3a(q, k) + b(q, k)].$$

Here k (p) denote the incoming (outgoing) momenta in the center-of-mass frame, $ME = 3k^2/4 - (1/a_2^t)^2$ is the total energy, and the kernel $K(p, q)$ is given by

$$K(p, q) = \frac{1}{2pq} \ln \left(\frac{q^2 + pq - p^2 - ME}{q^2 - pq - p^2 - ME} \right), \quad (10)$$

The amplitude $a(p, k)$ is normalized such that $a(k, k) = (k \cot \delta - ik)^{-1}$ with δ the elastic scattering phase shift. Furthermore, we have introduced a momentum cutoff Λ in the integral equations. Eqs. (8, 9) have previously been derived using a different method [20]. In the limit $\Lambda \rightarrow \infty$ these equations do not have a unique solution because the phase of the asymptotic solution is undetermined [21]. This is the same feature exhibited by the three-boson systems [15]. For a finite Λ this phase is fixed and the solution is unique. However, the equations with a cutoff display a strong cutoff dependence that does not appear in any order in perturbation theory. The amplitude $a(p, k = \text{const.})$ shows a strongly oscillating behavior. Varying the cutoff Λ slightly changes the asymptotic phase by a number of $\mathcal{O}(1)$ and results in large changes of the amplitude at the on-shell point $a(p = k)$. This cutoff dependence is not created by divergent Feynman diagrams. It is a nonperturbative effect and appears although all individual diagrams are superficially UV finite.

In order to understand this strong Λ dependence, it is useful to note that the integral equations (8, 9) are $SU(4)$ symmetric in the UV. Therefore it is sufficient to consider the $SU(4)$ limit ($a_2^t = a_2^s = a_2$), since the cutoff dependence is a problem rooted in the UV behavior of the amplitudes. Furthermore, we note that the equations for $a_+ = [a + b]$ and $a_- = [a - b]$ decouple in the $SU(4)$ limit. The equations for a_+ and a_- are

$$\frac{3}{4} \left(1/a_2 + \sqrt{3p^2/4 - ME} \right)^{-1} a_+(p, k) \quad (11)$$

$$= 2K(p, k) + \frac{2}{\pi} \int_0^\Lambda \frac{q^2 dq}{q^2 - k^2 - i\epsilon} 2K(p, q) a_+(q, k)$$

$$\frac{3}{4} \left(1/a_2 + \sqrt{3p^2/4 - ME} \right)^{-1} a_-(p, k) \quad (12)$$

$$= -K(p, k) - \frac{2}{\pi} \int_0^\Lambda \frac{q^2 dq}{q^2 - k^2 - i\epsilon} K(p, q) a_-(q, k).$$

The equation for a_+ is exactly the same equation as in the case of spinless bosons while the equation for a_- is the same equation as in the $S = 3/2$ channel. The $S = 3/2$ equation is well behaved and its solution is very insensitive to the cutoff [14]. Consequently, the cutoff dependence in Eqs. (8, 9) stems solely from the equation for a_+ . As an example, the cutoff dependence of $a_+(p, k = 0)$ is shown by the solid, dashed, and dash-dotted curves in Fig. 3 for three different cutoffs, $\Lambda = 1.0, 2.0, 3.0 \times 10^4 a_2^{-1}$.

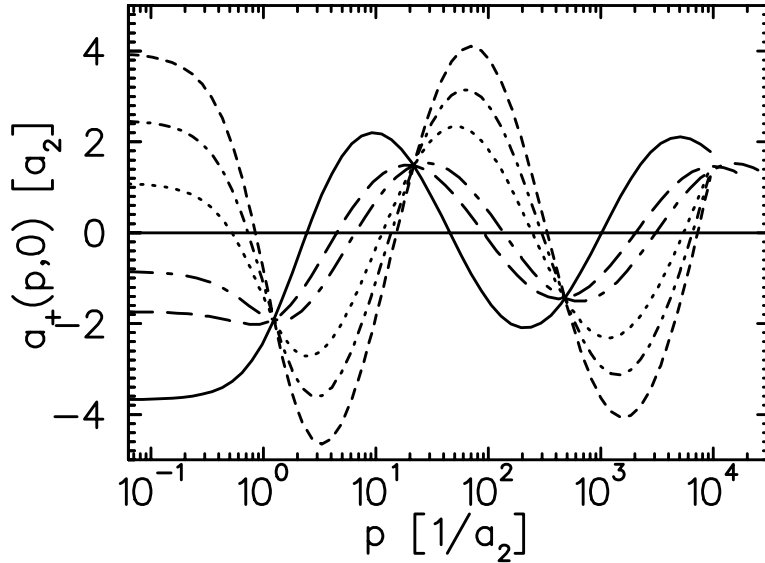


Figure 3: Cutoff dependence of $a_+(p, k = 0)$. Solid, dashed, and dash-dotted curves are for $H = 0$ and $\Lambda = 1.0, 2.0, 3.0 \times 10^4 a_2^{-1}$, respectively. Dotted, short-dash-dotted, and short-dashed curves show the effect of the three-body force for $\Lambda = 10^4 a_2^{-1}$ and $H = -6.0, -2.5, -1.8$, respectively.

Dependence on the cutoff reflects incorrect renormalization, and thus intolerable dependence on higher-energy modes. We need to ensure that only low-energy modes contribute explicitly, otherwise the power counting used to order two-body interactions—and in particular to produce Eq. (5) in lowest order regardless of the regularization procedure—would not hold. Now, since the equation for a_+ is the same as in the boson case, we already know the solution to this problem: a one-parameter three-body force counterterm $H(\Lambda)/\Lambda^2$ that runs with the cutoff Λ [15]. We simply replace

$$K(p, k) \rightarrow K(p, k) + \frac{H(\Lambda)}{\Lambda^2} \quad (13)$$

in Eq. (11). The dotted, short-dash-dotted, and short-dashed curves in Fig. 3 show the effect of the three-body force $H(\Lambda)$ on $a_+(p, k = 0)$ for $\Lambda = 10^4 a_2^{-1}$ and $H = -6.0, -2.5, -1.8$. It is clearly seen that the variation of H for a constant Λ has the

same effect on the amplitude as varying the cutoff. Consequently, we can compensate the changes in the asymptotic phase when Λ is varied by adjusting the three-body force term appropriately. (A more rigorous discussion of the renormalization procedure can be found in Ref. [15].)

We can obtain an approximate expression for the running of $H(\Lambda)$ from invariance under the renormalization group. Requiring that the equation for a_+ does not change its form when the high momentum modes are integrated out, we find

$$H(\Lambda) = -\frac{\sin(s_0 \ln(\Lambda/\Lambda_*) - \arctg(1/s_0))}{\sin(s_0 \ln(\Lambda/\Lambda_*) + \arctg(1/s_0))} \quad (14)$$

where $s_0 \approx 1.0064$ and Λ_* is a dimensionful parameter that determines the asymptotic phase of the off-shell amplitude [15]. The running of the three-body force $H(\Lambda)$ according to Eq. (14) is shown by the solid line in Fig. 4. The dots are obtained by adjusting $H(\Lambda)$

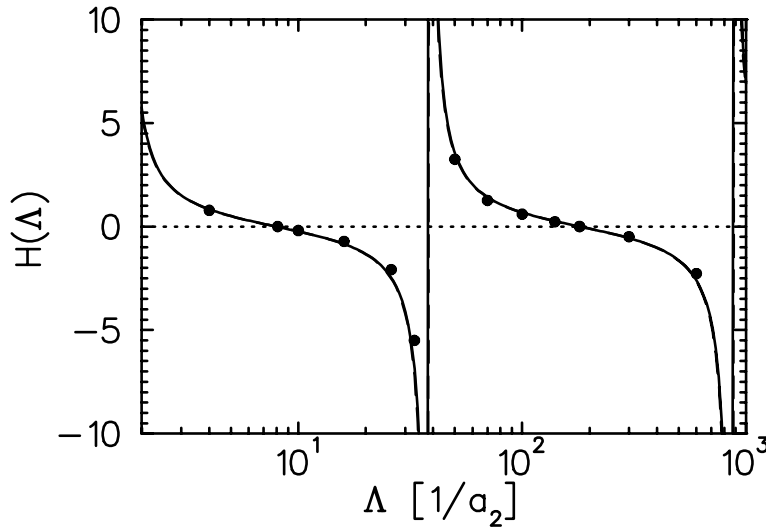


Figure 4: Running of $H(\Lambda)$ for $\Lambda_* = 0.9 \text{ fm}^{-1}$: (a) from Eq. (14) (solid line), (b) from numerical solution of Eq. (11) (dots).

such that the low-energy solution of Eq. (11) remains unchanged when Λ is varied. The observed agreement provides a numerical justification for our procedure. The three-body force is periodic with $H(\Lambda_n) = H(\Lambda)$ for $\Lambda_n = \Lambda \exp(n\pi/s_0) \approx \Lambda(22.7)^n$. In particular, the bare three-body force vanishes for a discrete set of cutoffs. Note, however, that invariance under continuous changes in the cutoff does require a non-vanishing bare three-body force.

Formally, this three-body force term is obtained by adding

$$\mathcal{L}_3 = -\frac{2MH(\Lambda)}{\Lambda^2} \left(g_T^2 N^\dagger (\vec{T} \cdot \vec{\sigma})^\dagger (\vec{T} \cdot \vec{\sigma}) N \right) \quad (15)$$

$$\begin{aligned}
& + \frac{1}{3} g_T g_S \left[N^\dagger (\vec{T} \cdot \vec{\sigma})^\dagger (\mathbf{S} \cdot \boldsymbol{\tau}) N + h.c. \right] \\
& + g_S^2 N^\dagger (\mathbf{S} \cdot \boldsymbol{\tau})^\dagger (\mathbf{S} \cdot \boldsymbol{\tau}) N \Big),
\end{aligned}$$

to the Lagrangian (2). Eq. (15) represents a contact three-body force written in terms of dibaryon and nucleon fields. Via a Gaussian path integration it is equivalent to a true three-nucleon force,

$$\begin{aligned}
\mathcal{L}_3 = & - \frac{2MH(\Lambda)}{\Lambda^2} \left(\frac{g_T^4}{4\Delta_T^2} (N^T \tau_2 \sigma_k \sigma_2 N)^\dagger (N^\dagger \sigma_k \sigma_l N) (N^T \tau_2 \sigma_l \sigma_2 N) \right. \\
& + \frac{1}{3} \frac{g_T^2}{2\Delta_T} \frac{g_S^2}{2\Delta_S} \left[(N^T \tau_2 \sigma_k \sigma_2 N)^\dagger (N^\dagger \sigma_k \tau_l N) (N^T \sigma_2 \tau_l \tau_2 N) + h.c. \right] \\
& \left. + \frac{g_S^4}{4\Delta_S^2} (N^T \sigma_2 \tau_k \tau_2 N)^\dagger (N^\dagger \tau_k \tau_l N) (N^T \sigma_2 \tau_l \tau_2 N) \right).
\end{aligned} \tag{16}$$

By performing a Fierz rearrangement, it can then be shown that the three terms in Eq. (16) are equivalent. As a consequence, there is only one three-body force which is also $SU(4)$ symmetric. Therefore the choice of the three-body force, Eq. (15), is by no means arbitrary. In fact, it is the only S -wave three-body force with no derivatives that can be written down. This can be seen by remembering that three fermions (antifermions) in a completely symmetric spatial wave need to be in a completely antisymmetric spin-isospin state which is a $\bar{\mathbf{4}}(\mathbf{4})$ of $SU(4)$. Consequently, the three-body force transforms as $\mathbf{4} \otimes \bar{\mathbf{4}} = \mathbf{1} \oplus \mathbf{15}$, but only the singlet is separately invariant under the spin and isospin subgroups. The two-body parameters g_T^2/Δ_T and g_S^2/Δ_S appear in Eq. (15) because the effect of the three-body force cannot be separated from the effect of the two-body interaction. Both are linked by the renormalization procedure. Naive power counting would suggest that the three-nucleon force scales with $1/(Mm_\pi^4)$. The three-body force from Eqs. (15, 16), however, is enhanced by the renormalization group flow by two powers of a_2 . Using Eq. (6), it is found to scale as $a_2^2/(Mm_\pi^2)$ which makes it leading order. $H(\Lambda)$ contains one new dimensionful parameter, Λ_* , which must be calculated from QCD or determined from experiment.

Recently Mehen, Stewart, and Wise [22] remarked that the two-nucleon amplitude (5) is approximately $SU(4)$ symmetric for $p \gg 1/a_2$. They also noticed that the only S -wave four-nucleon force that can be written down is $SU(4)$ symmetric. Furthermore, there are no contact interactions with more than four nucleons without derivatives because of the Pauli principle. It is also reasonable to assume that the low-energy dynamics of other nuclei is dominated by S -wave interactions, as is the case for the deuteron and the triton. Therefore our $SU(4)$ symmetric three-body force together with the findings of Mehen *et al.* [22] gives an explanation for the approximate $SU(4)$ symmetry [17] in nuclei.

Now we are in position to solve the full equations for the broken $SU(4)$ case. Including the three-body force from above into Eqs. (8, 9), we have

$$\frac{3}{2} \left(1/a_2^t + \sqrt{3p^2/4 - ME} \right)^{-1} a(p, k) = K(p, k) + \frac{2H(\Lambda)}{\Lambda^2} \tag{17}$$

$$\begin{aligned}
& + \frac{2}{\pi} \int_0^\Lambda \frac{q^2 dq}{q^2 - k^2 - i\epsilon} \left[K(p, q)[a(q, k) + 3b(q, k)] + \frac{2H(\Lambda)}{\Lambda^2}[a(q, k) + b(q, k)] \right] \\
& 2 \frac{\sqrt{3p^2/4 - ME} - 1/a_2^s}{p^2 - k^2} b(p, k) = 3K(p, k) + \frac{2H(\Lambda)}{\Lambda^2} \\
& + \frac{2}{\pi} \int_0^\Lambda \frac{q^2 dq}{q^2 - k^2 - i\epsilon} \left[K(p, q)[3a(q, k) + b(q, k)] + \frac{2H(\Lambda)}{\Lambda^2}[a(q, k) + b(q, k)] \right].
\end{aligned} \tag{18}$$

We need one three-body datum to fix the three-body force parameter Λ_* . We choose the experimental value for the $S = 1/2$ nd scattering length, $a_3^{(1/2)} = (0.65 \pm 0.04)$ fm [23], and find $\Lambda_* = 0.9$ fm $^{-1}$. (For the special cutoffs with vanishing $H(\Lambda)$, we recover the results of Ref. [24].) Although one three-body datum is needed as input, the EFT has not lost its predictive power. We can still predict (i) the energy dependence of $S = 1/2$ nd scattering and (ii) the binding energy of the triton.

The resulting energy dependence of $S = 1/2$ nd scattering for three different cutoffs is shown in Fig. 5. It is clearly seen that the introduction of the three-body force renders

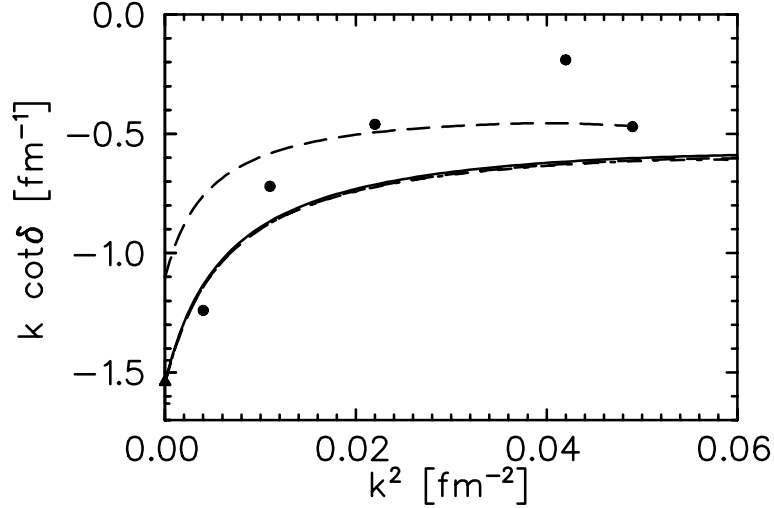


Figure 5: Energy dependence for $S = 1/2$ nd scattering with three different cutoffs $\Lambda = 1.9, 6.0, 11.6$ fm $^{-1}$ (hidden under the solid curve) for $\Lambda_* = 0.9$ fm $^{-1}$. Dashed curve gives estimate of range corrections. Data are from the phase-shift analysis of van Oers and Seagrave [25] (dots) and a measurement by Dilg *et al.* [23] (triangle).

the low-energy amplitude cutoff independent. The scattering length is reproduced exactly because it was used to fix Λ_* . The agreement for finite momentum is at least encouraging. Our experience from the $S = 3/2$ channel is that the range corrections improve the agreement considerably [14]. The dashed curve in Fig. 5 gives a crude estimate of these corrections. In our calculations we take the deuteron binding energy B_2 from experiment

and determine a_2^t from Eq. (7). The dashed curve in Fig. 5 is obtained by taking the experimental value of a_2^t as input and leaving all other parameters unchanged. From the estimated size of the range corrections, we anticipate an improved agreement once these corrections are included. (One should also keep in mind that the experimental phase-shift analysis [25] does not give any error estimate.)

The triton binding energy is obtained from the solution of the homogeneous equations corresponding to Eqs. (17, 18) for $E = -B_3$. In Fig. 6 we show the bound state spectrum as a function of the cutoff Λ for a particular Λ_* . The shallowest bound state is the triton.

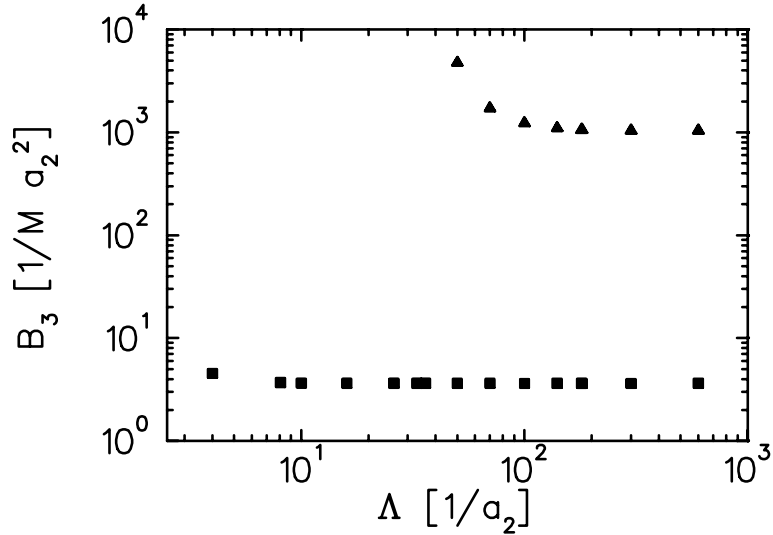


Figure 6: Three-nucleon bound state spectrum for $\Lambda_* = 0.9 \text{ fm}^{-1}$. The shallowest bound state corresponds to the triton.

Its binding energy is cutoff independent as long as $\Lambda \gg 1/a_2$. However, as Λ is increased new deeper bound states appear whenever $H(\Lambda)$ goes through a pole. These new bound states appear with infinite binding energy directly at the pole. As the cutoff is further increased, their binding energy decreases and becomes cutoff independent as well. The poles of $H(\Lambda)$ can be parametrized as

$$\Lambda_n = \underbrace{f(\Lambda_* a_2)}_{\mathcal{O}(1)} \exp(n\pi/s_0) a_2^{-1}. \quad (19)$$

One counts n bound states for $\Lambda_{n-1} < \Lambda < \Lambda_n$. However, only the states between threshold and $\Lambda \sim m_\pi \sim 1/R$ are within the range of the EFT. By solving Eq. (19) for the number of bound states n , we then obtain

$$\#_{BS} = \frac{s_0}{\pi} \ln \left(\frac{a_2}{R} \right) + \mathcal{O}(1), \quad (20)$$

and recover the well-known Efimov effect [11]. In the limit $a_2 \rightarrow \infty$, an infinite number of shallow three-body bound states accumulates at threshold. That these bound states are shallow follows from the fact that the binding energy is naturally given in units of $1/(Ma_2^2)$ which vanishes as $a_2 \rightarrow \infty$. Furthermore, we also recover the Thomas effect [12]. In a hypothetical world where $\Lambda \sim 1/R \rightarrow \infty$, the range of the EFT increases and deeper and deeper physical bound states appear. Consequently, there is an infinitely deep bound state for $\Lambda \rightarrow \infty$. However, for $m_\pi \sim 1/R$ as in the real world the deep bound states are outside the range of the EFT and their presence does not influence the physics of the shallow ones.

Moreover, the variation of the parameter Λ_* gives a natural explanation for the Phillips line [10]. The three-nucleon system has been studied with many phenomenological two-nucleon potentials where particles can have momenta as high as 1 GeV. Different phase-equivalent two-body potential models have different off-shell amplitudes, which are equivalent to different three-body forces. In some cases explicit three-body potentials have also been added. These models can be viewed as particular examples of high-energy dynamics. At low energies, they must all be equivalent to the EFT with particular values for its parameters. What we have shown is that in leading order they can only differ in the value of the *single* three-body parameter Λ_* . Varying Λ_* then generates a *line*. In Fig. 7 we show the Phillips line obtained in the EFT compared with results from various potential model calculations [26] and the experimental values for B_3 and $a_3^{(1/2)}$. Our Phillips line is slightly below the one from the potential models. We expect this discrepancy to be reduced once range corrections are taken into account.

The dynamics of QCD chooses a particular value of Λ_* , which up to higher order corrections is $\Lambda_* = 0.9 \text{ fm}^{-1}$. For this value we find $B_3 = 8.0 \text{ MeV}$ for the triton binding energy, to be compared with the experimental result $B_3^{exp} = 8.48 \text{ MeV}$, which is known to very high precision. The theory without pions seems to work for triton physics. However, to draw definite conclusions one has to include the range corrections for both the energy dependence and the triton binding energy.

The three-body force (16) involves three nucleons at the same point (within the resolution of the EFT) in a relative S wave. It cannot contribute to the $S = 3/2$ channel where the three nucleons spin in the same direction. Indeed, we have found that the quartet can be well predicted with two-nucleon input alone [14]. We stress that the three-body force that arises at low energies is not the same that appears at higher energies. As momentum increases, part of the three-body contact operator (16) may be resolved into iterations of the two-body potential. The strong renormalization of this operator at low energies is not necessarily in contradiction with the potential-model-based phenomenology where three-body forces are found to be small.

4 Conclusions

We have studied the three-nucleon system using EFT methods. While for nd scattering in the $S = 3/2$ channel precise predictions are obtained in a straightforward way [14], the

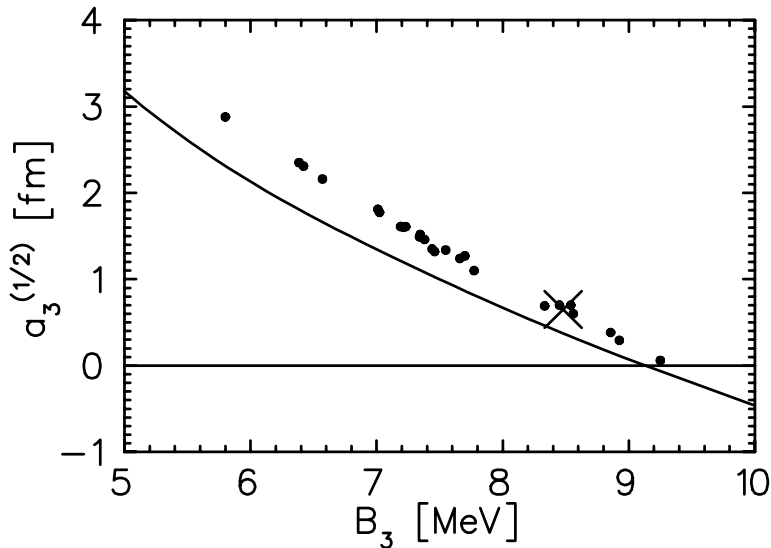


Figure 7: $S = 1/2$ nd scattering length $a_3^{(1/2)}$ as function of the triton binding energy B_3 (Phillips line): EFT to leading order (solid line); potential models (dots); and experiment (cross).

$S = 1/2$ channel is more complicated. It displays a strong cutoff dependence even though all individual diagrams are UV finite. In this channel a nonperturbative renormalization takes place similar to the case of spinless bosons. This renormalization group argument requires an $SU(4)$ symmetric three-body force which is enhanced to leading order. Together with the recent observations of Mehen *et al.* [22] this gives an explanation for the approximate $SU(4)$ symmetry in nuclei. Furthermore, we find that the Phillips line is a consequence of variations in the new dimensionful parameter Λ_* which is introduced by the three-body force. Λ_* is not determined by low-energy two-nucleon data alone and has to be determined from a three-body datum. In the appropriate limits for the two-body parameters a_2 and R , we also recover the well known Thomas and Efimov effects.

The leading order of the EFT gives a quantitative description of the triton binding energy and the energy dependence for $S = 1/2$ nd scattering. The theory shows the potential for a realistic description of the triton once range corrections are included. Immediate applications of the EFT include polarization observables in nd scattering and triton properties such as its charge form factor. The incorporation of the long-range Coulomb force would widen the possible applications considerably as pd scattering and the physics of ^3He becomes accessible. Work on these extensions is in progress.

Finally, since in leading order the pionless theory is equivalent to a theory with explicit (perturbative) pions [6], the success in the three-nucleon system opens the possibility of applying the EFT method to a large class of systems with three or more nucleons.

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